PLAY WITH MATH

TEST NO-03 TIME:-3Hrs.

F.M:-100

General instructions:-

1. All questions are compulsory.
2. This question paper contains 29 questions.
3. Questions 1-4 in section A are short-answer type questions carrying 1 mark each.
4. Questions 5-12 in section B are short-answer type questions carrying 2 marks each.
5. Questions 13-23 in section C are long-answer type questions carrying 4 marks each.
6. Questions 24-29 in section D are long-answer type questions carrying 6marks each.

Section-A

1.Find the vector of magnitude 7 in the direction of $\vec{a} = 3\hat{\imath} - 2\hat{\jmath} + 6\hat{k}$.

2. If A is a skew- symmetric matrix of order 3×3 then find the value of det(A).

For what value of k, the matrix $\begin{bmatrix} k & 2 \\ 3 & 4 \end{bmatrix}$ has no inverse .

3. Evaluate $\int_0^{\pi/4} secx$(secx +tanx) dx .

4. find the equation of plane passing through the point (1,0,-2) and normal to $\hat{\imath}+\hat{\jmath}-\hat{k}$.

Section-B

5. $f(x) = \begin{cases} \frac{2^{x+2}-16}{4^x-16}, & x \neq 2 \\ k & , x = 2 \end{cases}$ is continuous at x =2 then find K.

6.If $x^m y^n = (x+y)^{m+n}$, find $\frac{dy}{dx}$.

7. verify Roll's theorm for $f(x) = x^3 - 6x^2 + 11x - 6$ in [1,3].

8. A ladder 5m long is leaning against a wall . the bottom of the ladder s pulled along the ground ,away from the wall ,at the rate of 2cm/sec .how fast is its height on the wall decreasing when the foot of the ladder is 4 m away from the wall?

9. Using differentials find the approximate value of $\sqrt{25.3}$

10 . Evaluate : $\int \frac{logx}{(1+logx)^2} dx$

Or,

$\int \frac{dx}{9x^2+6x+5}$

11. Find a unit vector perpendicular to the plane of two vector $\vec{a} = \hat{\imath} - \hat{\jmath} + 2\hat{k}$ and $\vec{b} = 2\hat{\imath} + 3\hat{\jmath} - \hat{k}$.

Or,

Find the area of the parallegram having digonals.

$(3\hat{\imath} + \hat{\jmath} - 2\hat{k})$ and , $(\hat{\imath} - 3\hat{\jmath} + 4\hat{k})$

12. probability for solving specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively if both try to solve the problem independently ,find the probability that the problem is solved.

Or,

If A and B are two independent events such that P(A′) =0.65 P(A∪B) =0.65 and P(B) = p , find the value of p.

Section –C

13. Find the equation of the plane which contains the line of intersection of the planes $\hat{r} \cdot (\hat{\imath} + 2\hat{\jmath} + 3\hat{k}) - 4 = 0$, $\hat{r} \cdot (2\hat{\imath} + \hat{\jmath} - \hat{k}) + 5 = 0$ and which is the perpendicular to the plane $\hat{r} \cdot (5\hat{\imath} + 3\hat{\jmath} - 6\hat{k}) + 8 = 0$

14. Find the value of λ if four points with position vector $3\hat{\imath} + 6\hat{\jmath} + 9\hat{k}$, $\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$, , $2\hat{\imath} + 3j + 3\hat{k}$ and $4\hat{\imath} + 6\hat{\jmath} + \lambda\hat{k}$ are coplanar .

Or,

If $\vec{\alpha} = 3\hat{\imath} - \hat{\jmath}$,$\vec{\beta} = 2\hat{\imath} - \hat{\jmath} - 3\hat{k}$ then express $\vec{\beta}$ in the form $\vec{\beta}_1 + \vec{\beta}_2$, where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$.

15. prove :- $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right) = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x$

Or,

Solve :- $\tan^{-1}\frac{1-x}{1+x} = \frac{1}{2}\tan^{-1}x$, x > 0.

16. If x = a(cost +tsint) & y = a(sint –tcost) ,find $\frac{d^2y}{dx^2}$.

17. $\int_0^{\pi} \frac{sinx+cosx}{9+1 \quad sin2x} dx$

Or,

$\int_0^{\pi/2} \frac{x}{sinx+cosx}\, dx$

18. If a,b,c are positive and unequal ,show that value of the determinant

$\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is negative

19.Find the interval in which the function f(x) = $\frac{4x^2+1}{x}$ (x≠0) is

(i) increasing (ii) decreasing.

20.Show that the curves 2x = y^2 and 2xy=k cut at right angle if k^2 =8.

21. In answering a question on a multiple choice test ,a student either knows the answer or guesses . let $\frac{3}{4}$ be the probability that he know the answer and $\frac{1}{4}$ be the probability that he guesses . Assuming that a student who guesses the answer will be correct with probability $\frac{1}{4}$. what is the probability that the student knows the answer given that he answered it correctly ?

22. A pair of die thrown 6 times . if getting total of 9 is consider a success ,what is the probability of at least 5 successes ?

23.Solve the differential equation (x.logx)$\frac{dy}{dx}$ + y = $\frac{2}{x}$ log x

Section-D

24. Use product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the system of equation

x-y+2z =1 , 2y -3y =1 , 3x -2y +4z =2 .

25. A manufacturing company makes two model A and B of a product each pieces of model a required 9 labour hours for fabricating and 1labour hour for finishing .each pieces of model B requires 12 labour hours for fabricating and 3 labour for finishing For fabricating and finishing , the maximum labour hours available are 180 and 30 respectively . the company make a profit of rs 8000 on each pieces of model A and and rs 12000 on each pieces of model B how many pieces of model A and model B should be manufactured per week to realize a maximum profit.

26.Find the vector equation of the line passing through the point (1,2,-4) and perpendicular to the two line $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ & $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$

Or,

Find the distance between the point (6,5,9) and the plane determined by the points A(3,-1,2) , B(5,2,4) ,c(-1,-1,6)

27. An open tank with a sqare base vertical sides is to be constructed from a metal sheet so as to hold a given quantity of water .show that the cost of the material will be least when the depth of the tank is half of its width.

28. find the area of the region enclosed between the two circle $x^2+y^2=4$ and $(x-2)^2+y^2=4$ Or,

Find the area lying above x-axis and and included between parabola $y^2=4x$ and the circle $x^2+y^2=8x$.

29. prove that the volume of the largerst cone that can be inscribed in a sphere of radius R is is $\frac{8}{27}$ of the volume of the sphere.

Or,

Let f: N→R ,be a function defined as $f(x) = 4x^2+12x+15$. Show that ,f:N→S ,where s, is the range of f , is invertible. Also ,find the inverse of f.